Solve the following differential equations. **SCORE:** _____ **/ 30 PTS**

[a] $\frac{dr}{d\theta} = \frac{r(r^2-3\theta^2)}{\theta(r^2-\theta^2)}$ **FINAL ANSWER:** $\theta^6 e^{\frac{r^2}{\theta^2}} = Cr^2$

8 POINTS

[b] $(x^2+tx^4)dt + x\,dx = 0$ **FINAL ANSWER:** $x^{-2} = -t - \frac{1}{2} + Ce^{2t}$

13 POINTS

[c] $(1-x^2\tan y)dy + 2x\,dx = 0$ **FINAL ANSWER:** $x^2\cos y + \sin y = C$

10 POINTS

[a] $\underbrace{\theta(r^2-\theta^2)}_{M}dr + \underbrace{r(3\theta^2-r^2)}_{N}d\theta = 0$

$M(tr,t\theta) = t\theta(t^2r^2-t^2\theta^2) = t^3\theta(r^2-\theta^2) = t^3M(r,\theta)$

$N(tr,t\theta) = tr(3t^2\theta^2-t^2r^2) = t^3r(3\theta^2-r^2) = t^3N(r,\theta)$

HOMOGENEOUS

$r = v\theta$ $\quad dr = v\,d\theta + \theta\,dv$

$\theta(v^2\theta^2-\theta^2)(v\,d\theta+\theta\,dv) + v\theta(3\theta^2-v^2\theta^2)d\theta = 0$

$(\cancel{v^3\theta^3} - v\theta^3 + 3v\theta^3 - \cancel{v^3\theta^3})d\theta + (v^2\theta^4-\theta^4)dv = 0$

$2v\cancel{\theta^3}d\theta + \theta^4(v^2-1)dv = 0$

$\int \frac{1}{\theta}d\theta = \int \frac{1-v^2}{2v}dv = \int\left(\frac{1}{2}v^{-1} - \frac{1}{2}v\right)dv$

$\ln|\theta| = \frac{1}{2}\ln|v| - \frac{1}{4}v^2 + C$

$4\ln|\theta| = 2\ln\left|\frac{r}{\theta}\right| - \left(\frac{r}{\theta}\right)^2 + C$

$\theta^4 = \frac{Cr^2}{\theta^2}e^{-\frac{r^2}{\theta^2}}$

$\theta^6 e^{\frac{r^2}{\theta^2}} = Cr^2$

OR

$\underline{\Theta = vr}$ $\underline{d\Theta = v\,dr + r\,dv}$

$$\underline{vr(r^2 - v^2r^2)\,dr + r(3v^2r^2 - r^2)(v\,dr + r\,dv) = 0}$$

$$(vr^3 - v^3r^3 + 3v^3r^3 - vr^3)\,dr + (3v^2r^4 - r^4)\,dv = 0$$

$$\underline{2v^3\cancel{r^3}\,dr + r^{\cancel{4}}(3v^2 - 1)\,dv = 0}$$

$$\underline{\int \frac{1}{r}\,dr = \int \frac{1-3v^2}{2v^3}}\,dv = \int\left(\tfrac{1}{2}v^{-3} - \tfrac{3}{2}v^{-1}\right)dv$$

$$\underline{\ln|r| = -\tfrac{1}{4}v^{-2} - \tfrac{3}{2}\ln|v| + C}$$

$$\underline{4\ln|r| = -\left(\frac{\Theta}{r}\right)^{-2} - 6\ln\left|\frac{\Theta}{r}\right| + C}$$

$$r^4 = \frac{Cr^6}{\Theta^6}e^{-\frac{r^2}{\Theta^2}}$$

$$\underline{\Theta^6 e^{\frac{r^2}{\Theta^2}} = Cr^2}$$

ALTERNATE SOLUTION FOR [a]

☆ USE ONLY 1 VERSION FOR GRADING

OR $(\theta r^2 - \theta^3)\,dr + (3\theta^2 r - r^3)\,d\theta = 0$

2ND ALTERNATE SOLUTION FOR [a]

☆ USE ONLY 1 VERSION FOR GRADING

GUESS $\mu = \theta^a r^b$

$$\underbrace{(\theta^{a+1} r^{b+2} - \theta^{a+3} r^b)}_{M}\,dr + \underbrace{(3\theta^{a+2} r^{b+1} - \theta^a r^{b+3})}_{N}\,d\theta = 0$$

$$M_\theta = (a+1)\theta^a r^{b+2} - (a+3)\theta^{a+2} r^b$$

$$N_r = 3(b+1)\theta^{a+2} r^b - (b+3)\theta^a r^{b+2}$$

$$a+1 = -(b+3)$$

$$-(a+3) = 3(b+1)$$

$$-2 = 2b \longrightarrow b = -1$$

$$a = -3$$

$$\mu = \theta^{-3} r^{-1}$$

$$\underbrace{(\theta^{-2} r - r^{-1})}_{M}\,dr + \underbrace{(3\theta^{-1} - \theta^{-3} r^2)}_{N}\,d\theta = 0$$

$M_\theta = -2\theta^{-3} r = N_r$ EXACT ✓

$$f = \int (\theta^{-2} r - r^{-1})\,dr$$

$$= \tfrac{1}{2}\theta^{-2} r^2 - \ln|r| + C(\theta)$$

$$f_\theta = \cancel{-\theta^{-3} r^2} + C'(\theta) = 3\theta^{-1} \cancel{- \theta^{-3} r^2}$$

$$C(\theta) = 3\ln|\theta|$$

$$\tfrac{1}{2}\theta^{-2} r^2 - \ln|r| + 3\ln|\theta| = C$$

$$\frac{\theta^3}{r} e^{\frac{r^2}{2\theta^2}} = C$$

$$\theta^3 e^{\frac{r^2}{2\theta^2}} = Cr$$

NO PARTIAL CREDIT IF YOUR CHECKPOINT WAS MISSING, INCORRECT OR BOGUS

[b] $\frac{dx}{dt} + x = -tx^3$ BERNOULLI

$v = x^{1-3} = x^{-2}$

$\frac{dv}{dt} = -2x^{-3}\frac{dx}{dt}$

$-2x^{-3}\frac{dx}{dt} - 2x^{-2} = 2t$

OR

$x = v^{-\frac{1}{2}}$

$\frac{dx}{dt} = -\frac{1}{2}v^{-\frac{3}{2}}\frac{dv}{dt}$

$-\frac{1}{2}v^{-\frac{3}{2}}\frac{dv}{dt} + v^{-\frac{1}{2}} = -tv^{-\frac{3}{2}}$

$\frac{dv}{dt} - 2v = 2t$

$\mu = e^{\int -2\,dt} = e^{-2t}$

CHECK: $\frac{d}{dt}e^{-2t} = -2e^{-2t}$ ✓

$e^{-2t}\frac{dv}{dt} - 2e^{-2t}v = 2te^{-2t}$

NO PARTIAL CREDIT IF YOUR CHECKPOINT WAS MISSING INCORRECT OR BOGUS

$e^{-2t}v = \int 2te^{-2t}\,dt + C$

$= -te^{-2t} - \frac{1}{2}e^{-2t} + C$

u		dv
$2t$	$+$	e^{-2t}
2	$-$	$-\frac{1}{2}e^{-2t}$
0		$\frac{1}{4}e^{-2t}$

$v = -t - \frac{1}{2} + Ce^{2t}$

$x^{-2} = -t - \frac{1}{2} + Ce^{2t}$

[C] $M = 2x \qquad N = 1 - x^2 \tan y$

$M_y = 0 \qquad N_x = -2x \tan y$

$$\frac{N_x - M_y}{M} = \frac{-2x\tan y}{2x} = -\tan y$$ FUNCTION OF ONLY y

$$\mu = e^{\int -\tan y\, dy} = e^{\ln|\cos y|} = \cos y$$

↳ $u = \cos y$

$du = -\sin y\, dy$

$$\underbrace{(\cos y - x^2 \sin y)}_{N} dy + \underbrace{2x\cos y}_{M}\, dx = 0$$

$M_y = -2x\sin y = N_x$ EXACT ✓

$$f = \int 2x\cos y\, dx = x^2\cos y + C(y)$$

$$f_y = -\cancel{x^2\sin y} + C'(y) = \cos y - \cancel{x^2\sin y}$$

$$C(y) = \sin y$$

$$f = x^2\cos y + \sin y = C$$

NO PARTIAL CREDIT
IF YOUR CHECKPOINT
WAS MISSING, INCORRECT OR BOGUS

OR

ALTERNATE SOLUTION FOR [C]

★ USE ONLY 1 VERSION FOR GRADING

$$\underline{\frac{dx}{dy} - \frac{1}{2}x\tan y = -\frac{1}{2}x^{-1}}$$

BERNOULLI

$$\underline{v = x^{1--1} = x^2}$$

$$\frac{dv}{dy} = 2x\frac{dx}{dy}$$

$$\underline{2x\frac{dx}{dy} - x^2\tan y = -1}$$

OR

$$x = v^{\frac{1}{2}}$$

$$\frac{dx}{dy} = \frac{1}{2}v^{-\frac{1}{2}}\frac{dv}{dy}$$

$$\underline{\frac{1}{2}v^{-\frac{1}{2}}\frac{dv}{dy} - \frac{1}{2}v^{\frac{1}{2}}\tan y = -\frac{1}{2}v^{-\frac{1}{2}}}$$

$$\underline{\frac{dv}{dy} - (\tan y)v = -1}$$

$$\underline{\mu = e^{\int -\tan y\,dy} = e^{\ln|\cos y|} = \cos y}$$

$$\underline{(\cos y)\frac{dv}{dy} - (\sin y)v = -\cos y}$$

$$\underline{\text{CHECK: } \frac{d}{dy}\cos y = -\sin y} \checkmark$$

$$\underline{(\cos y)v = \int -\cos y\,dy + C}$$

$$\underline{= -\sin y + C}$$

$$\underline{v = -\tan y + C\sec y}$$

$$\underline{x^2 = -\tan y + C\sec y}$$

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